Antisymmetrization effects in three-cluster systems

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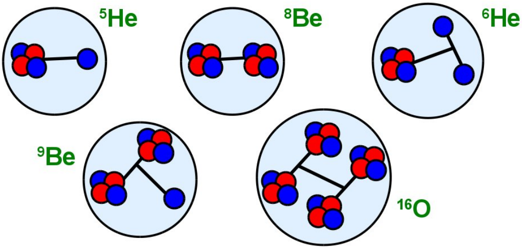
Quick look at cluster systems

The study of cluster systems has a long history and remains one of the main topics in nuclear physics (M. Freer, Rep. Prog. Phys. 70 (2007) 2149–2210)

(So) many examples : ^{5}He$=\alpha+n$, ^{8}Be$=\alpha+\alpha$, ^{6}He$=\alpha+n+n$, ^{9}Be$=\alpha+\alpha+n$, ^{16}O$=\alpha+\alpha+\alpha+\alpha$, ...



3-cluster systems have interesting features :
2-neutron/proton halo, borromean, Hoyle state ^{12}C

Common points :

- their wave function ψ extends to large distances
- only a few bound states
- continuum still poorly known

Topic of my Ph.D. thesis :
Study the continuum states of 3-cluster systems within a microscopic model

Microscopic Description of Cluster Systems

Microscopic Hamiltonian H

$$H_{mic} = \sum_{i=1}^{A} T_i + \sum_{i\leq j=1}^{A} V_{ij}$$

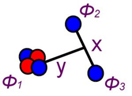
$\star V_{ij} = V_{NUCL}^{CENTRAL} + V_{NUCL}^{SO} + V_{COUL} + \cancel{V_{TENSOR}}$

$\star V_{NUCL}^{CENTRAL}$ = MINNESOTA potential : effective NN interaction, Gaussian shape, one adjustable parameter $u \sim 1$

$\star V^{SO}$: zero-range NN Spin-Orbit interaction

Microscopic Wave Function Ψ

$$\Psi_{RGM} = \mathcal{A}\left[\phi_1\phi_2\phi_3 \; g(\mathbf{x},\mathbf{y})\right]$$



- $\mathcal{A}$: Antisymmetrization operator (exchanges of nucleons between clusters)
- ϕ_i : internal cluster wave functions
- g : relative wave function ($\mathbf{x}$,$\mathbf{y}$ are Jacobi coordinates)

Generator Coordinate Method

Construction of Ψ within a microscopic cluster model ?

• (once more), the relative wave function g is expended on a Gaussian basis Γ but with two generator coordinates :

$$\Psi_{GCM} = \mathcal{A}\left[\phi_1\phi_2\phi_3 \sum_{ij} f(\mathbf{X}_i,\mathbf{Y}_j)\ \Gamma(\mathbf{X}_i)\Gamma(\mathbf{Y}_j)\right]$$

equivalent to :

$$\Psi_{GCM} = \sum_{ij} f(\mathbf{X}_i,\mathbf{Y}_j)\ \Phi(\mathbf{X}_i,\mathbf{Y}_j)$$

• Ψ_{GCM} is built as a linear combination of Slater determinants
$\Rightarrow$ very suitable for computer calculations

• generator function f depends on two coordinates

HOW TO DEAL WITH CONTINUUM STATES (R-MATRIX METHOD) WITH 2 GENERATOR COORDINATES ?

First Solution : 2-Body Collision With One Distorted Nucleus

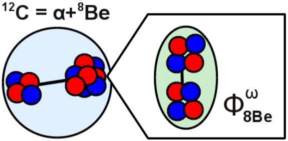
e.g. : Instead of seeing ^{12}C as a triple-α structure, one can rather imagine ^{12}C as an α+^{8}Be structure in which ^{8}Be has also an $\alpha + \alpha$ substructure :

$$H_{\alpha+\alpha}\,\phi^{\omega}_{\mathbf{8Be}} = E^{\omega}\,\phi^{\omega}_{\mathbf{8Be}} \quad \& \quad \phi^{\omega}_{\mathbf{8Be}} = \mathcal{A}\left[\phi_\alpha\phi_\alpha \sum_i f(\mathbf{X}_i)\Gamma(\mathbf{X}_i)\right]$$

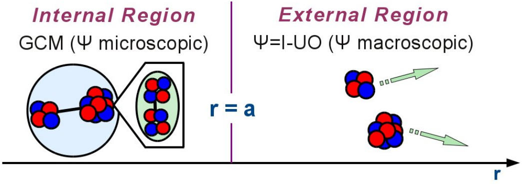
$$\Psi_{GCM} = \sum_{\omega}\mathcal{A}\left[\phi_\alpha\,\phi^{\omega}_{\mathbf{8Be}} \sum_j \tilde{f}_{\omega}(\mathbf{Y}_j)\,\Gamma(\mathbf{Y}_j)\right]$$

◦ As a first step, we solve the 2-cluster equation of ^{8}Be$= \alpha + \alpha$.
E^{ω} are the eigenenergies and ϕ^{ω} the related eigenfunctions (number ω =number X_i)

◦ As a second step, we introduce the eigenfunctions ϕ^{ω} into the second 2-cluster equation ^{12}C=^{8}Be+α



- the 2-generator coordinates problem is replaced by a multichannel 1-generator coordinate problem ☺
- physically, we study a 2-body collision in which one nucleus is distorted



What about the scattering states ?

One can use the Microscopic R-matrix Method in the multichannel 2-body case ☺

- The channel radius a is chosen so that the antisymmetrization between α and ^{8}Be can be neglected $\Rightarrow$ $a \sim 10$ fm
- the more we include eigenfunctions $\phi^{\omega}_{^8Be}$ (number $X_i \nearrow$), the better the "(1+2)-cluster" structure is well approximated

Second Solution : 3-Body Collision Using The Hyperspherical Formalism

Hyperspherical Formalism

Starting from the Jacobi coordinates x and y,
we define the hyperradius ρ and the hyperangle α as :

$$\rho = \sqrt{|x|^2 + |y|^2} \;\; (range\ [0, \infty]) \quad \& \quad \alpha = atan(|y|/|x|) \;\; (range\ [0, \pi/2])$$

$\longrightarrow$ the new set $(\rho, \alpha, \Omega_x, \Omega_y)$ forms the set of hypershperical coordinates

Hypermomentum K and Hyperspherical Harmonics $\mathcal{Y}_K$

In the hyperspherical formalism, the role of L^2 and $Y_\ell(\Omega)$:

$$\begin{aligned} \Psi(\mathbf{r}) &= u_\ell(r)\ Y_\ell(\Omega) & [2-body\ problem] \\ L^2\ Y_\ell(\Omega) &= \ell(\ell+1)\ Y_\ell(\Omega) \end{aligned}$$

are generalized as :

$$\begin{aligned} \Psi(\mathbf{x}, \mathbf{y}) &= \chi_K(\rho)\ \mathcal{Y}_K(\alpha, \Omega_x, \Omega_y) & [3-body\ problem] \\ K^2\ \mathcal{Y}_K(\alpha, \Omega_x, \Omega_y) &= K(K+4)\ \mathcal{Y}_K(\alpha, \Omega_x, \Omega_y) \end{aligned}$$

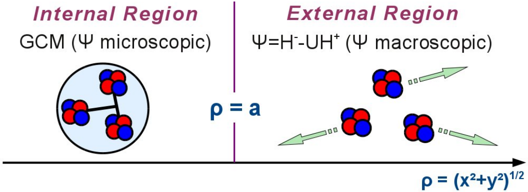
$\longrightarrow$ new quantum number K and new harmonics $\mathcal{Y}_K$

GCM in the hypersherical formalism. What's it like ?

The wave function Ψ is decomposed in K-partial waves.
The generator hyperradius R is used as a unique variational coordinate ⌣̈ :

$$\Psi_{GCM} = \sum_{K}^{Kmax} \mathcal{A}\left[\phi_{\alpha 1}\phi_{\alpha 2}\phi_{\alpha 3} \ \sum_{i} \hat{f}_K(R_i)\ G_K(R_i)\right]$$

The summation over K is truncated in practice



What about the scattering states ?

The hyperspherical R-matrix version (very recent, exact 3-body collision !) ⌣̈

- the channel radius a is chosen as a certain value of $\rho = \sqrt{|x|^2 + |y|^2}$
  Antisymmetrization in term of ρ is less intuitive ⌢̈
- the phase shifts δ depend on the number of K-partial waves

Eigenvalues of $\mathcal{A}$

How to evaluate antisymmetrization effects ? (hypershperical formalism)

remember, the $R(a)$ matrix is defined at such distance that the antisymmetrization on Ψ can be neglected.

$\hookrightarrow$ One can evaluate the action of the operator $\mathcal{A}$ by solving the eigenvalue equation :

$$\mathcal{A}\left[\phi_1\phi_2\phi_3\chi_j^{J^\pi}(R)\right] = \mu_j^{J^\pi} \; \phi_1\phi_2\phi_3\chi_j^{J^\pi}(R)$$

- The eigenvalues must be range between 0 and 1 :

$$0 \leq \mu_1 \leq \mu_2 \leq \ldots \mu_N \leq 1$$

- The eigenstates χ_j related to a $\mu_j = 0$ eigenvalue are FORBIDDEN STATES (Pauli principle, arises when $R \rightarrow 0$)
- An eigenvalue $\mu_j = 1$ means that $\mathcal{A}$ has no action

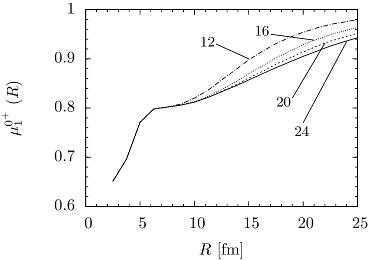
Antisymmetrization effects on ^{6}He nucleus

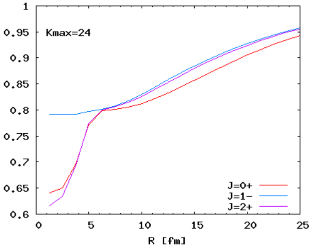
What happens in the ^{6}He case ?

- The lowest eigenvalue $\mu_1^{0^+}$ curves do not converge until K_{max} is fixed to 24.
- $\mu_1^{J^\pi}$ curves are greater than 0.95 for $R > 25$ fm.

Conclusion ?

• With the discretization $R = 1.25, 2.5, \ldots, 25$ fm (20 points), the size of the hypersherical basis is reasonable.
• The second solution can be applied on ^{6}He$=\alpha + n + n$ system :
A.Damman & P.Descouvemont, Phys. Rev. C80 (2009) 044310





The phase shifts also depend strongly on K_{max} value

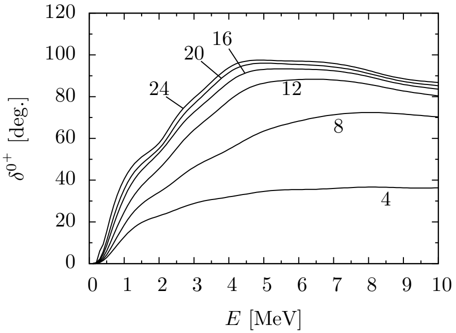
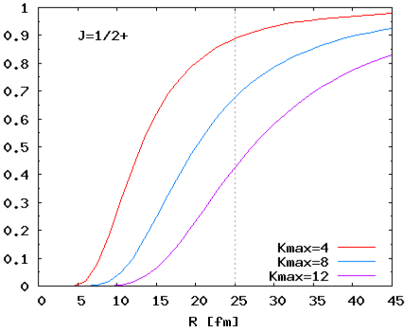
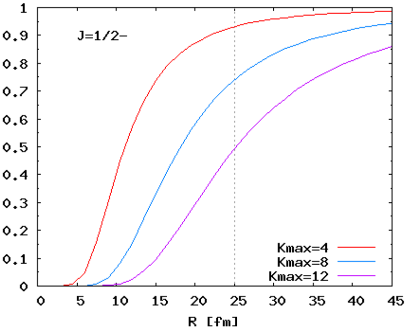


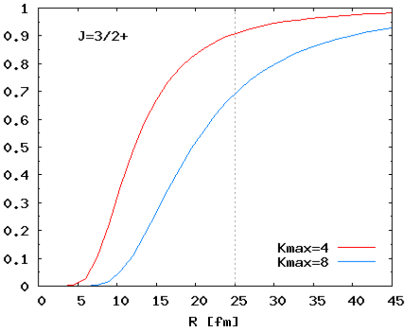
Figure: Convergence the resonant phase shift δ^{0^+} for ^{6}He with respect to K_{max}

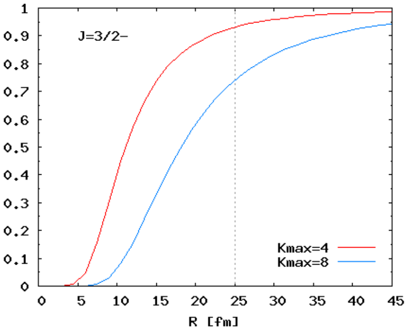
Antisymmetrization effects on ^{9}Be nucleus

It's clear that we can't apply the hypershperical formalism on the $^{9}\text{Be}=\alpha+\alpha+n$ system $\longrightarrow$ We must use the distorded nucleus technique

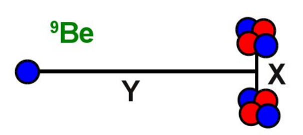






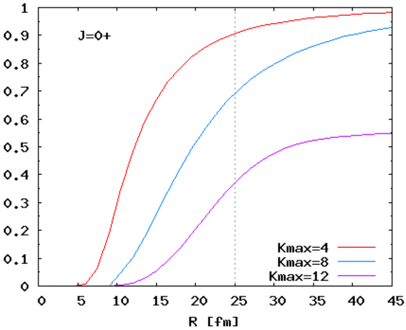


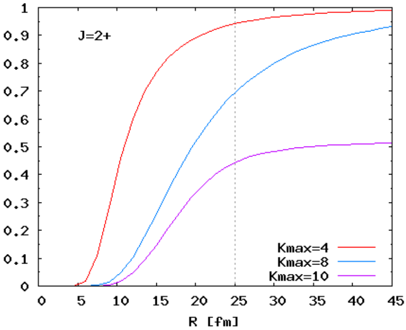
Intuitively, even for a great value of R, the both α can stay close to each other ($R = \sqrt{|\alpha \leftrightarrow \alpha|^2 + |n \leftrightarrow \alpha + \alpha|^2}$) and $\mathcal{A}$ can not be neglected at all



Antisymmetrization effects on ^{12}C nucleus

The same conclusion applies to the $^{12}\mathrm{C}=\alpha+\alpha+\alpha$ system $\longrightarrow$ We must use the distorded nucleus technique





Conclusion

1. What's my Job ?
   - Investigate the 3-cluster continuum states
2. How can I do that ?
   - by combining a microscopic model (GCM) and the R-matrix Method (MRM)
3. more specifically ? What about 3-cluster systems ?
   - 2 ways : Distorded nucleus (1+2-body) / Hyperspherical formalism (3-body)
4. How can I chose between them ?
   - by looking at the antisymmetrization.
   - chose the second way when it's possible $\longrightarrow$ ($\checkmark$) ^{6}He ,($\times$) ^{9}Be,($\times$) ^{12}C
5. Is that all ? What's next ?
   - compute the phase shifts of ^{9}Be and ^{12}C using the first way

Merry Christmas

&

Happy New Year